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Performance Evaluation of Adaptive Two-Dimensional Turbo Product Codes Composed of Hamming Codes

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***Abstract* - In this paper, the asymptotic performance of two-dimensional (2-D) turbo product codes composed of Hamming codes are provided. A performance evaluation of various adaptive two-dimensional turbo product codes (7,4,3)× (7,4,3), (15,11,3) ×(15,11,3), (31,26,3) ×(31,26,3), (63,57,3) ×(63,57,3), (127,120,3) × (127,120,3) is presented. Simulation results demonstrate that in high SNR regions, turbo product codes outperform all other code performance as selecting proper adaptive parameters and test patterns in the decoding algorithm.**

Index Terms -Turbo product codes, encoding process, code construction, asymptotic performance, BER

I. INTRODUCTION

During the past sixty years, Forward Error Correction (FEC) schemes have been a popular technique that gained much interests from communication engineers. This coding research was initiated by Hamming, Golay, and others in the late 1940s. In the 1950s and 1960s, many researchers focused primarily on developing the theory of efficient encoders and decoders. In 1970, the first book entitled “An Introduction to Error-Correction Codes” was published, covering both block and convolutional codes [1].

Since turbo codes were invented in 1993 and employed in the third generation mobile communications systems, turbo product codes (TPCs) also called block turbo codes as well as Low-Density Parity-Check (LDPC) Codes have been extensively studied and applied [2][3][4][5][6][7][8]. In fact, they have similar concatenated construction.

Turbo codes have excellent performance at low signal-to-noise ratios (SNR), which are very close to the Shannon limit [2]. However, in some cases they may have very low minimum distances, despite of fairly large interleaver lengths. This results in error floor imposed by the minimum distance.

Turbo product codes are a special scheme of serial concatenation, in which there exists no interleaver/ deinterleaver, since their construction have natural row and column interleaving process. The main interesting point concerns their minimum distance, which is equal to the product of the minimum distances of the component row and column subcodes. The decoder of 2-D TPCs employs soft input/soft output iterated decoding algorithm [3]. By iterating the process in a turbo-like approach, very good performance can be obtained in high SNR regions.

The asymptotic performances of TPCs are basically concentrated on 2-D TPCs composed of extended Hamming codes [9]. In this paper, we choose Hamming codes as TPC component codes and calculate their asymptotic performance.

Furthermore, one of the main contributions in this paper lies in the fact that we employ decoding algorithms addressed in [4], and obtain excellent performance in high SNR by properly selecting adaptive parameters, i.e., test pattern parameter p, weighting factor α and reliability factor β.

The rest of the paper is organized as follows: Section II gives code performance of the two-dimensional turbo product codes composed of Hamming codes. Section III describes simulated performance of adaptive 2-D turbo product codes. Section IV concludes this paper.

II. The code performance of 2-D TPCs composed of Hamming codes

A. The encoding process and code construction

The encoding process of TPCs is shown in Fig. 1, where two kinds of encoding sequence are shown respectively.

Given two binary block code $C^1(n_1,k_1)$, $C^2(n_2,k_2)$, where k_i and n_i (i=1,2) denote the information bit and codeword, respectively, the 2-D TPC can be expressed as

$$C_{tpc} = C^1(n_1,k_1)\times C^2(n_2,k_2) = (n_1\times n_2, k_1\times k_2) \qquad (1)$$

which is obtained by writing the information bits in a matrix $(k_2\times k_1)$, encoding the k_2 rows by row encoder $C^1(n_1,k_1)$ that appends (n_1-k_1) parity check bits to each row, and then encoding the n_1 columns by the column encoder $C^2(n_2,k_2)$ that appends (n_2-k_2) parity check bits to each column. Therefore, all the matrix rows are codewords of row encoder C^1 and all the columns are codewords of C^2.

Similarly, if C^1 denotes column encoder, C^2 denotes row encoder, all the matrix rows are codewords of row encoder C^2 and all the columns are codewords of C^1. As an example, the code (7,4) × (7,4) construction is illustrated in Fig. 2.

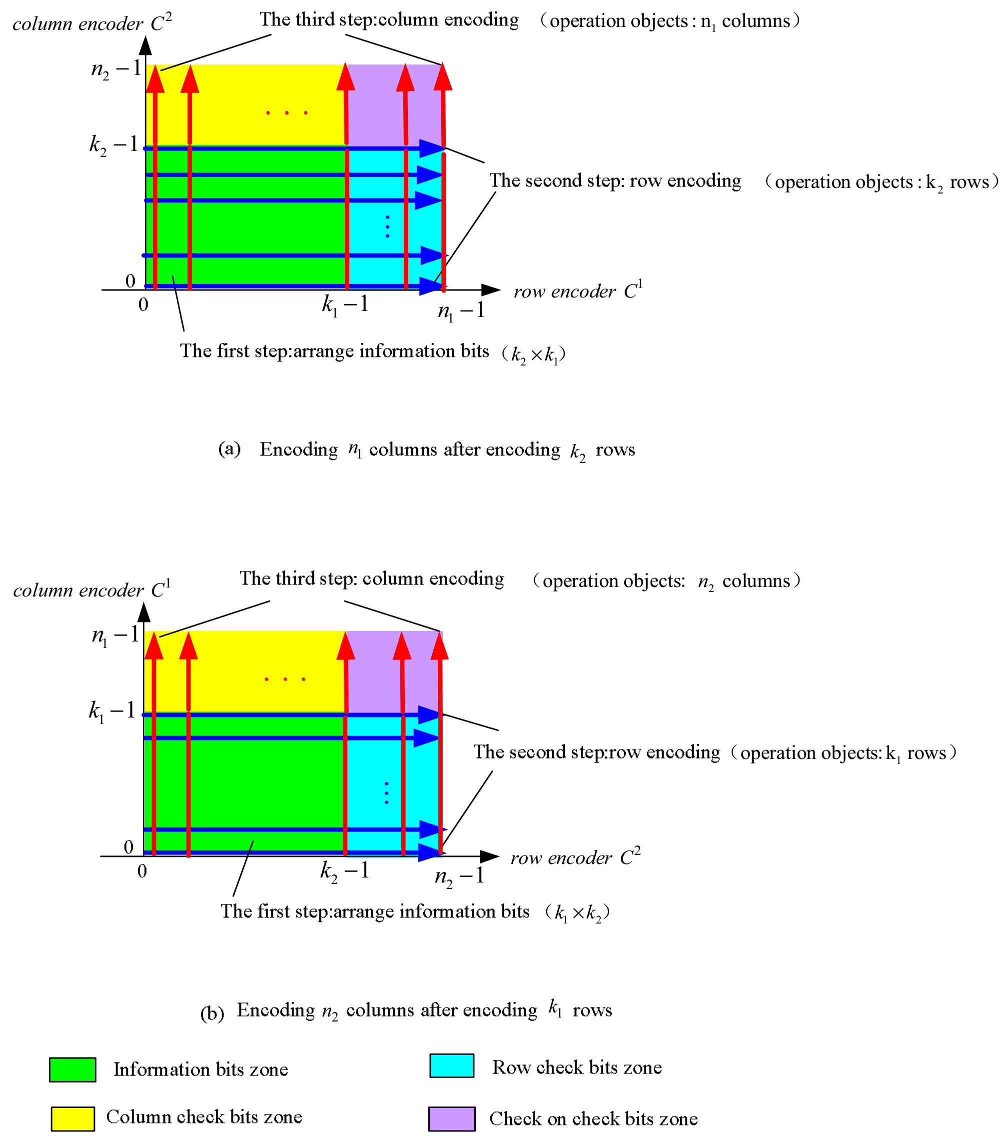

Fig. 1 The encoding process

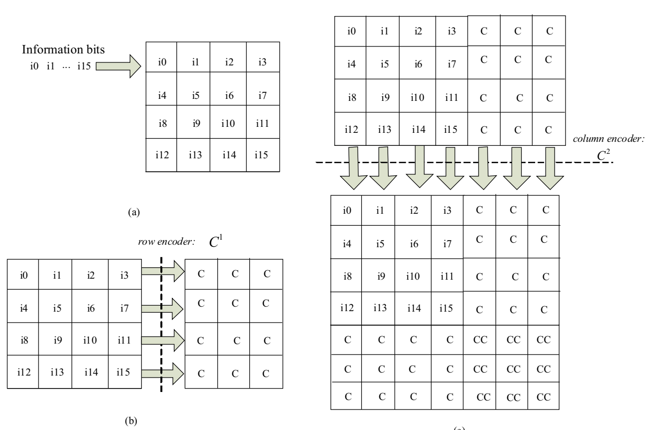

Fig. 2 The code (7,4)×(7,4) construction example

B. The decoding process

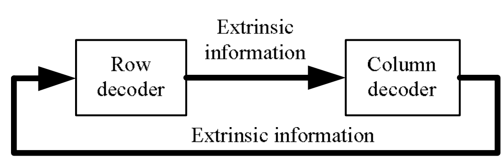

Fig. 3 Schematic diagram of the decoding process

A typical 2-D TPC decoder consists of a row decoder and a column decoder which work iteratively one after the other, as shown in Fig. 3. The iteration process continues until the maximum number of iterations has been reached or convergence has taken place.

C. The asymptotic performances of 2-D TPCs composed of Hamming codes

In [9], the asymptotic performance of extended Hamming product codes are calculated and ploted. In this section, we will introduce the analysis of the asymptotic performance of 2-D TPCs composed of two identical Hamming codes. To calculate the asymptotic performance, the minimum distance $d_{\min}$ and the weight enumerator A_i for a Hamming code are used.

For a Hamming code (7,4), $d_{\min}$ =3, the weigh enumerator $A_3(7,4)$ can be computed in the following formula [10]:

$$A_t = \frac{1}{n+1}\left\{ C_n^t + n(-1)^{\left\lfloor \frac{t+1}{2} \right\rfloor} C_{\left\lfloor \frac{n-1}{2} \right\rfloor}^{\left\lfloor \frac{t}{2} \right\rfloor} \right\} \tag{2}$$

where n is the code length, t is defined to be the number of codewords c in a binary linear code C having Hamming weight $w(c) = t$, $C_n^t = \frac{n!}{t!(n-t)!}$,and $\lfloor X \rfloor$ takes the nearest integer that is less than or equal to X.

Specifically,

$$A_3(7,4) = \frac{1}{8}[C_7^3 + 7(-1)^2 C_3^1] = 7 \tag{3}$$

$$A_3(15,11) = \frac{1}{16}[C_{15}^3 + 15(-1)^2 C_1^1] = 35 \tag{4}$$

$$A_3(31,26) = \frac{1}{32}[C_{31}^3 + 31(-1)^2 C_{15}^1] = 155 \tag{5}$$

$$A_3(63,57) = \frac{1}{64}[C_{63}^3 + 63(-1)^2 C_{31}^1] = 651 \tag{6}$$

$$A_3(127,120) = \frac{1}{128}[C_{127}^3 + 127(-1)^2 C_{63}^1] = 2667 \tag{7}$$

When a 2-D TPC composed of two identical Hamming codes is modulated by BPSK and transmitted over an AWGN channel , the maximum likelihood asymptote(MLA) can be written as [10][11]

$$P_{MLA} = \left(\frac{d_{\min}}{n}\right)^2 \left(A_{w_{\min}}(c)\right)^2 Q\left(\sqrt{\frac{2E_b}{N_0}\left(\frac{k}{n}\right)^2 (d_{\min})^2}\right)$$
$$= \frac{1}{2}\left(\frac{d_{\min}}{n}\right)^2 \left(A_{w_{\min}}(c)\right)^2 erfc\left(\frac{k}{n} d_{\min}\sqrt{\frac{E_b}{N_0}}\right) \tag{8}$$

Substituting (3)-(7) into (8), we can compute the asymptotic performance of 2-D TPCs composed of identical Hamming codes as shown in Fig. 4.

III. SIMULATION RESULTS OF ADAPTIVE TWO-DIMENSIONAL TURBO PRODUCT CODES DECODER

The adaptive 2-D TPCs decoder was firstly introduced by Pyndiah [3]. Different from the approach in [3] , in our study, the weight and reliability factors were fixed for each iteration as $\alpha = 0.5$, $\beta = 1$. Without loss of generality, we assume all-zero codeword is transmitted. The number of least reliable bits is chosen to be $p = 2$. Simulated performances are shown in Fig. 5-9. BER became zero after a certain Signal-to-Noise Ratio (SNR) was attained.

As seen from the curves, after the second iteration, when BER dropped to 0, for 2-D TPCs (7,4) ×(7,4), (15,11) ×(15,11), (31,26) × (31,26) (63,57) × (63,57), (127,120) ×(127,120) , the SNRs are 0.6dB, 2.8 dB, 4.9 dB, 7.6dB, 8.2dB respectively.

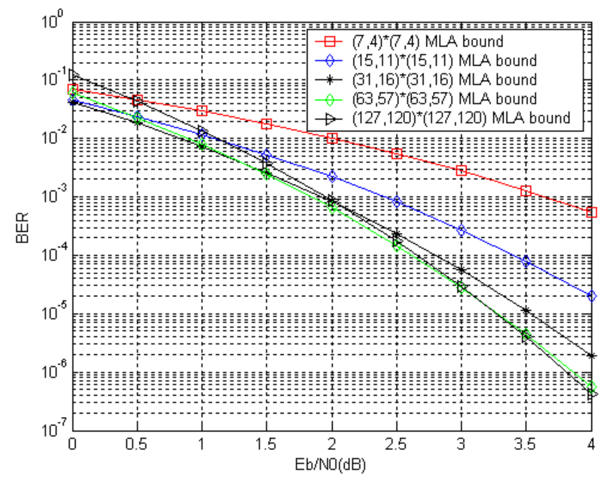

Fig. 4 The asymptotic performances of 2-D TPCs composed of identical Hamming codes

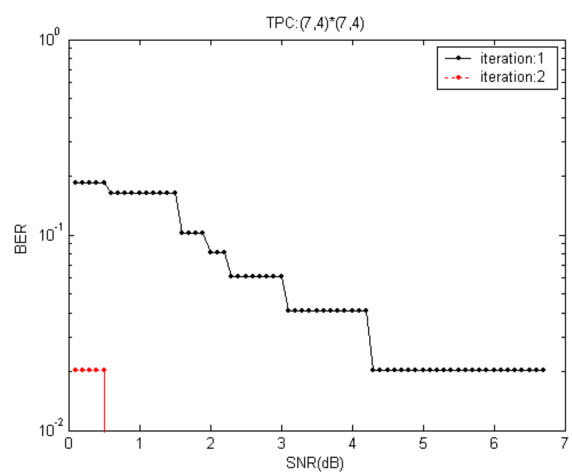

Fig. 5 BER performance for 2-D TPC (7,4) × (7,4)

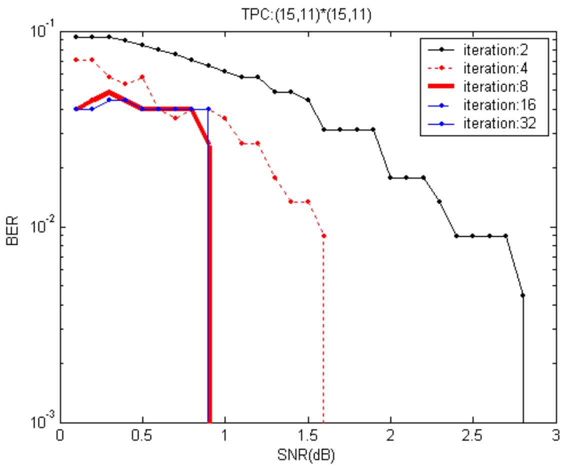

Fig. 6 BER performance for 2-D TPC (15,11) ×(15,11)

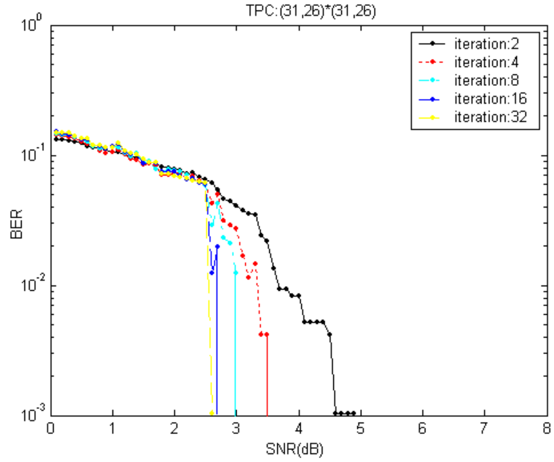

Fig. 7 BER performance for 2-D TPC (31,26) ×(31,26)

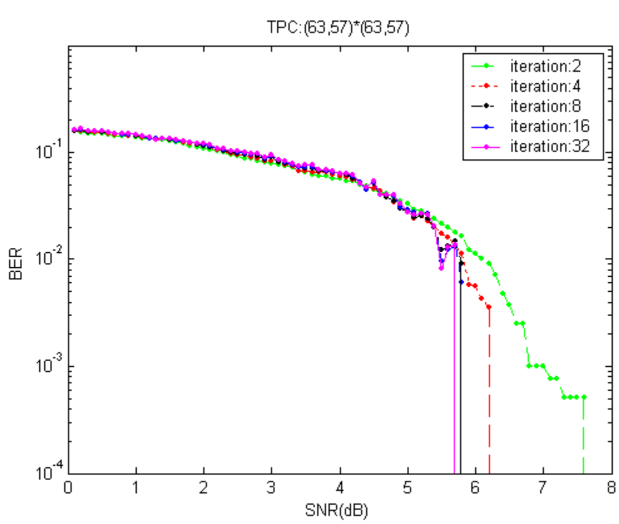

Fig. 8 BER performance for 2-D TPC (63,57) ×(63,57)

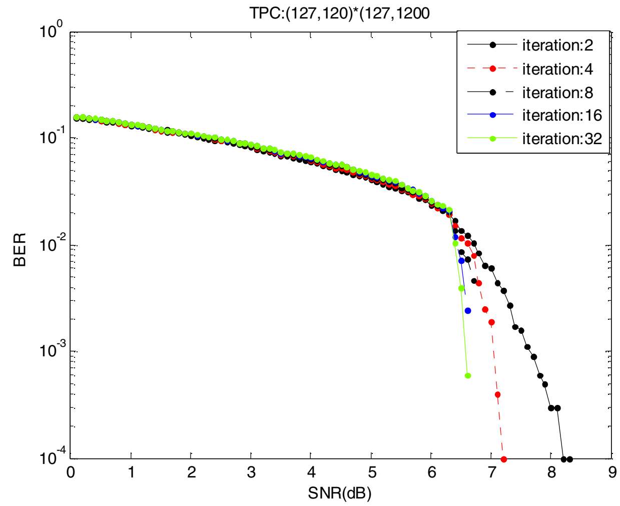

Fig. 9 BER performance for 2-D TPC (127,120) ×(127,120)

IV. Conclusions

In this paper, we computed the asymptotic performance of 2-D TPCs composed of Hamming codes using their weight distributions and the maximum likelihood asymptote (MLA). Iterative decoding of a TPC, which is performed by the suboptimum soft-in/soft-out decoding algorithm, shows relative good performance with low complexity through simulations. Specifically, BER of 2-D TPCs composed of Hamming codes is zero at high SNR regions. These results could also be obtained in 2-D TPCs composed of extended Hamming codes as selecting proper adaptive parameters [12]. Typical usage of these codes is in wireless applications requiring low error rates. The presented adaptive parameters allow designers and researchers to evaluate the code performance without inducing very long and usually impossible simulations. Therefore, turbo product codes can be useful for high rate data communications.

For future research, many interesting topics could be explored: 1) Bifurcation and chaos behavior for the proposed 2-D TPCs [13]; 2) Theoretical analysis on BER of zero in high SNR regions.

All the Matlab programs developed by the authors for obtaining the results proposed in this paper are available for any interested party at [14].

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References

[1] S. Lin, *An introduction to error error-correcting codes*, Englewood Cliffs, N.J. : Prentice Hall, 1970.

[2] C. Berrou, A. Glavieux and P. Thitimajshima, “Near Shannon limit error-correcting coding and decoding: Turbo codes(1),” *IEEE ICC’93*, vol 2/3, pp. 1064-1071, 1993.

[3] R. Pyndiah, Glavieux, A Picart, et al, “Near optimum decoding of product codes,” *IEEE GLOBECOM’94*, Vol.1, pp.339-343, 1994.

[4] R. Pyndiah, "Near optimum decoding of product codes: block turbo codes," *IEEE Trans. on Communications*,Vol. 46, No.8, pp.1003-1010, 1998.
[5] R. G. Gallager, "Low-density parity-check codes," *IRE Trans. on Information Theory*, pp.21-28, 1962.
[6] http://www.comtechefdata.com.cn/Products/Modems/Datasheets/ds-cdm700.pdf
[7] http://www.xilinx.com/ipcenter/catalog/logicore/docs/tpc_encoder.pdf.
[8] H. Song, Richard M. Todd and J. R. Cruz, "Applications of low-density parity-check cods to magnetic recording channels", *IEEE Journal on Selected Areas in Communications*, Vol. 19, No.5, pp.918-923, 2001.
[9] C. Franco and G. Roberto, "Extended Hamming product codes analytical performance evaluation for low error rate applications," *IEEE Trans. on Wireless Communications*, Vol.3, No.6, pp. 2353-2361, 2004.
[10] B.Stephen Wicker, Error Control Systems for Digital Communication and Storage, Prentice Hall, 1995.
[11] M. Ferrari, S. Bellini, "Importance sampling simulation of turbo product codes," *IEEE ICC'2001*,Vol.9, pp.2773-2777, 2001.
[12] Y. He, G. Zhu, B. Zhang et al, "The performance of STBC coded MIMO-OFDM system based on turbo product codes," *IEEE WCNM'2005*, Vol. 1, pp.653-656, 2005.
[13] Y. He, Francis C. M. Lau and C.K. Tse, "Study of dynamical behaviour of two-dimensional turbo product code decoders," to appear in *Chaos, Solitions and Fractals*, Published by Elsevier Science, Oxford,U.K.
[14] http://www.ee.cuhk.edu.hk/~yjhe: turbo product code research.